Benha University
Faculty of Engineering at Shoubra

ECE-508
Sensor Networks

Lecture #5
Topology Control

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Agenda

- Motivation, basics
- Power Control
- Backbone Construction
- Clustering
- Adaptive Node Activity

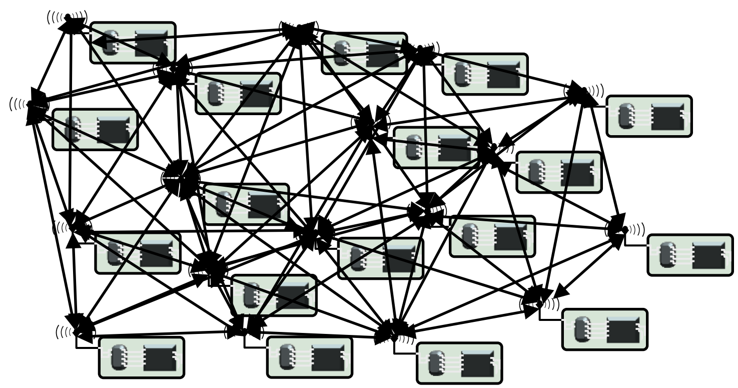
Goals of this chapter

- Networks can be too dense – too many nodes in close (radio) vicinity
- This chapter looks at methods to deal with such networks by
  - Reducing/controlling transmission power
  - Deciding which links to use
  - Turning some nodes off

MOTIVATION, BASICS

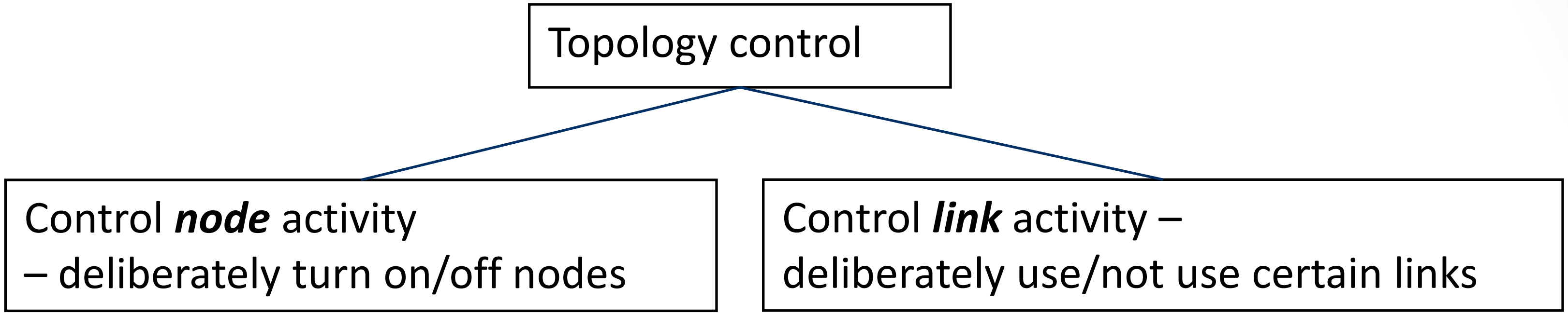
Motivation: Dense networks

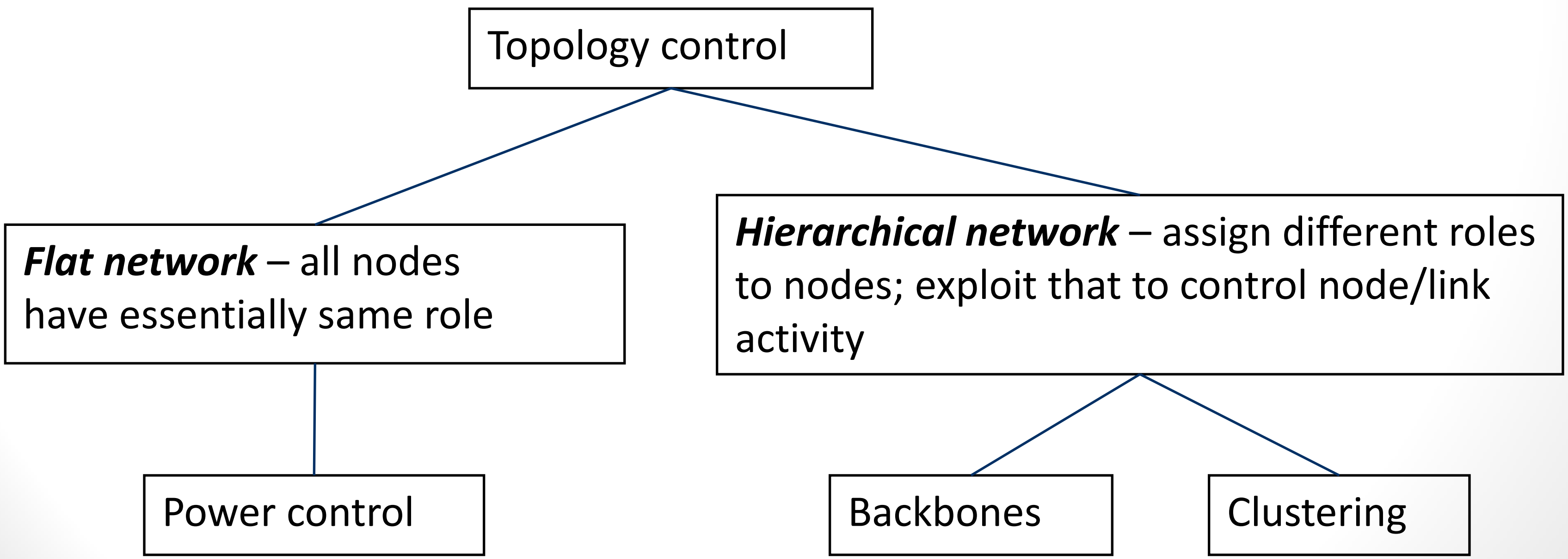
- In a very dense networks, too many nodes might be in range for an efficient operation
  - Too many collisions/too complex operation for a MAC protocol, too many paths to chose from for a routing protocol, …



- Idea: Make ***topology*** less complex
  - ***Topology***: Which node is able/allowed to communicate with which other nodes
  - Topology control needs to maintain invariants, e.g., connectivity

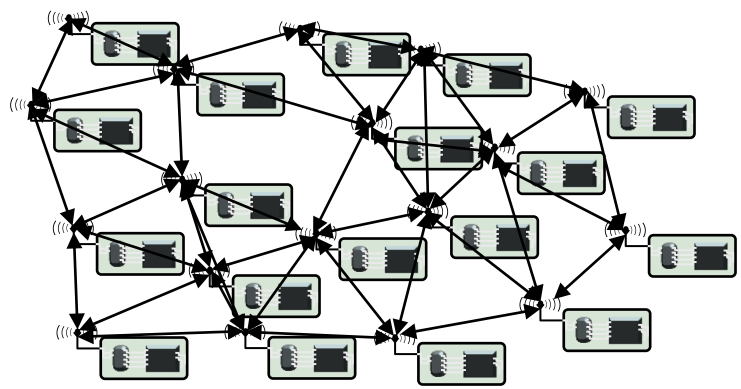
Options for topology control





Flat networks

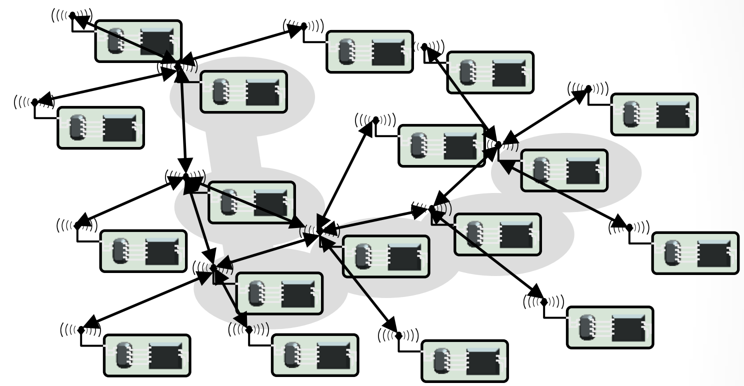
- Main option: Control transmission power
  - Do not always use maximum power
  - Selectively for some links or for a node as a whole
  - Topology looks “thinner”
  - Less interference, …



- Alternative: Selectively discard some links
  - Usually done by introducing hierarchies

Hierarchical networks – backbone

- Construct a ***backbone*** network
  - Some nodes “control” their neighbors – they form a (minimal) ***dominating set***
  - Each node should have a controlling neighbor
  - Controlling nodes have to be connected (backbone)
  - Only links within backbone and from backbone to controlled neighbors are used



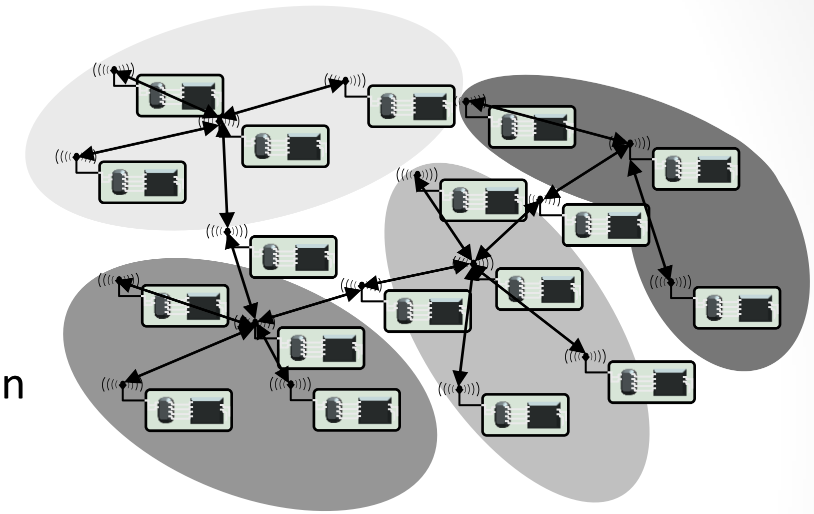
- Formally: Given graph G=(V,E), construct D ⊂ V such that

$$\forall v \in V : v \in D \vee \exists d \in D : (v,d) \in E$$

☺ Read it as : for all *v* **element of** V **such that** *v* **element of** D **or there exists** d **element of** D **such that** the **open interval** *v,d* **element of** E .

Hierarchical network – clustering

- Construct ***clusters***
  - Partition nodes into groups ("clusters")
  - Each node in exactly one group
    - Except for nodes "bridging" between two or more groups
  - Groups can have ***clusterheads***



- Typically: all nodes in a cluster are direct neighbors of their clusterhead
- Clusterheads are also a dominating set, but should be separated from each other – they form an ***independent set***
- Formally: Given graph G=(V,E), construct $C \subset V$ such that

$$\forall v \in V - C : \exists c \in C : (v,c) \in E$$

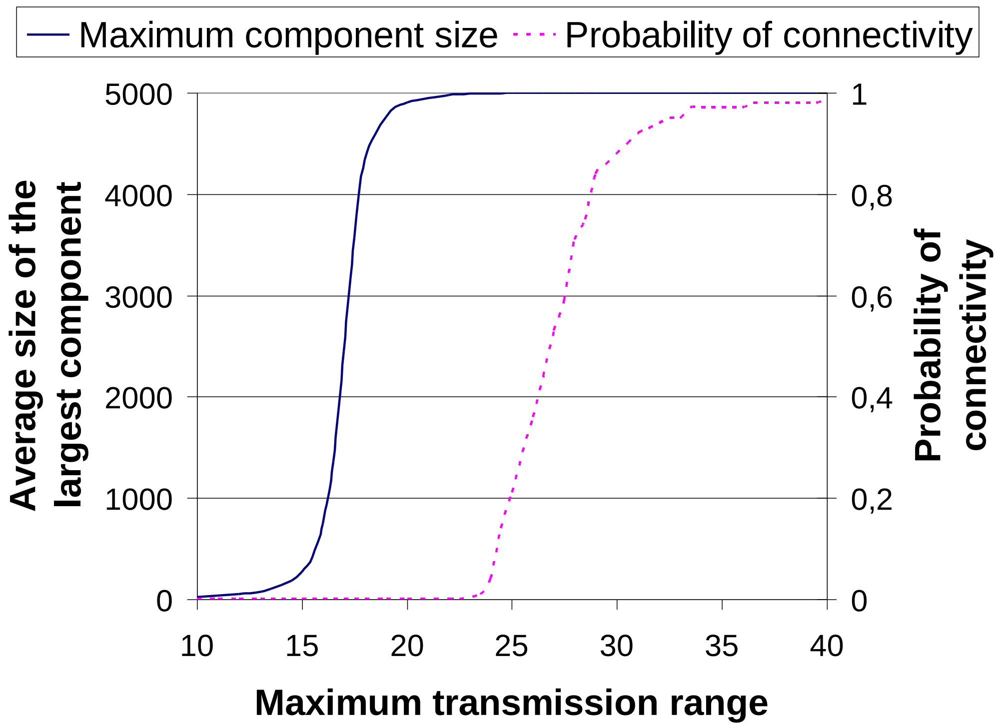
$$\forall c_1, c_2 \in C : (c_1, c_2) \notin E$$

Aspects of topology-control algorithms

- ***Connectivity*** – If two nodes connected in G, they have to be connected in G^0 resulting from topology control
- ***Stretch factor*** – should be small
  - ***Hop stretch factor***: how much longer are paths in G^0 than in G?
  - ***Energy stretch factor***: how much more energy does the most energy-efficient path need?
- ***Throughput*** – removing nodes/links can reduce throughput, by how much?
- Robustness to mobility
- Algorithm overhead

Example: Price for maintaining connectivity

- Maintaining connectivity can be very "costly" for a power control approach
- Compare power required for connectivity compared to power required to reach a very big maximum component



POWER CONTROL

Power control – magic numbers?

- Question: What is a good power level for a node to ensure "nice" properties of the resulting graph?
- Idea: Controlling transmission power corresponds to controlling the number of neighbors for a given node
- Is there an "optimal" number of neighbors a node should have?
  - Is there a "magic number" that is good irrespective of the actual graph/network under consideration?
- Historically, k=6 or k=8 had been suggested as such "magic numbers"
  - However, they optimize progress per hop – they do ***not*** guarantee connectivity of the graph!!

! Needs deeper analysis

Controlling transmission range

- Assume all nodes have identical transmission range r=r(|V|), network covers area A, V nodes, uniformly distr.
- Fact: Probability of connectivity goes to zero if:

$$r(|V|) \leq \sqrt{\frac{(1-\epsilon)A\log|V|}{\pi|V|}}, \text{ for any } \epsilon > 0$$

- Fact: Probability of connectivity goes to 1 for

$$r(|V|) \geq \sqrt{\frac{A(\log|V| + \gamma_{|V|})}{\pi|V|}}$$

  if and only if $\gamma_{|V|}$! 1 with |V|
- Fact (uniform node distribution, density ρ):

$$P(\text{G is } k\text{-connected}) \approx \left(1 - \sum_{l=0}^{k-1} \frac{(\rho\pi r^2)^l}{l!} e^{-\rho\pi r^2}\right)$$

Controlling number of neighbors

- Knowledge about range also tells about number of neighbors
  - Assuming node distribution (and density) is known, e.g., uniform
- Alternative: directly analyze number of neighbors
  - Assumption: Nodes randomly, uniformly placed, only transmission range is controlled, identical for all nodes, only symmetric links are considered
- Result: For connected network, required number of neighbors per node is **Θ (log |V|)**
  - It is ***not a constant***, but depends on the number of nodes!
  - For a larger network, nodes need to have more neighbors & larger transmission range! – Rather inconvenient
  - Constants can be bounded

Some example constructions for power control

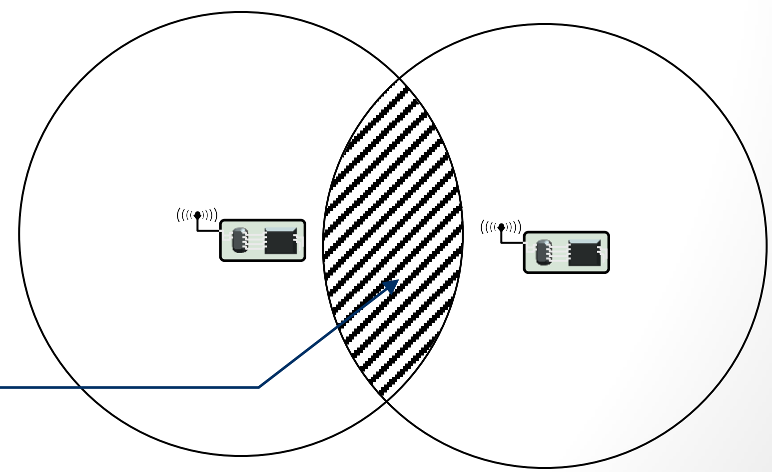
- Basic idea for most of the following methods:
  Take a graph G=(V,E), produce a graph G^0=(V,E^0) that maintains connectivity with fewer edges
  - Assume, e.g., knowledge about node positions
  - Construction should be local (for distributed implementation)

Example 1: Relative Neighborhood Graph (RNG)

- Edge between nodes u and v if and only if there is no other node w that is closer to either u or v
- Formally:

$$\forall u, v \in V : (u,v) \in E' \text{ iff}$$
$$\nexists w \in V : \max\{d(u,w), d(v,w)\} < d(u,v)$$

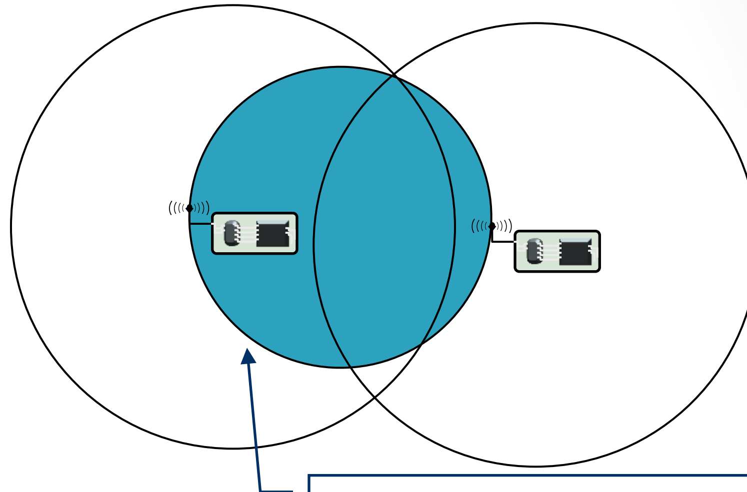
- RNG maintains connectivity of the original graph
- Easy to compute locally
- But: Worst-case spanning ratio is Ω (|V|)
- Average degree is 2.6



This region has to be empty for the two nodes to be connected

Example 2: Gabriel graph

- Gabriel graph (GG) similar to RNG
- Difference: Smallest circle with nodes u and v on its circumference must only contain node u and v for u and v to be connected



This region has to be empty for the two nodes to be connected

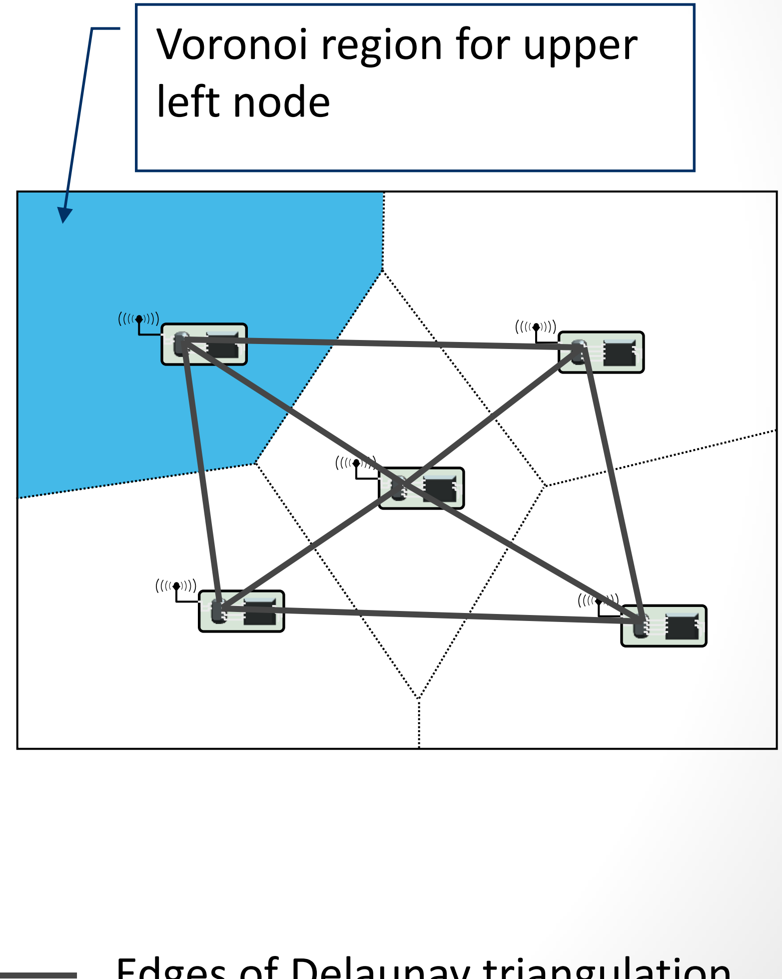
- Formally:

$$\forall u, v \in V : (u, v) \in E' \text{ iff}$$
$$\nexists w \in V : d^2(u, w) + d^2(v, w) < d^2(u, v)$$

- Properties: Maintains connectivity, Worst-case spanning ratio $\Omega(|V|^{1/2})$, energy stretch O(1) (depending on consumption model!), worst-case degree $\Omega(|V|)$

Example 3: Delaunay triangulation

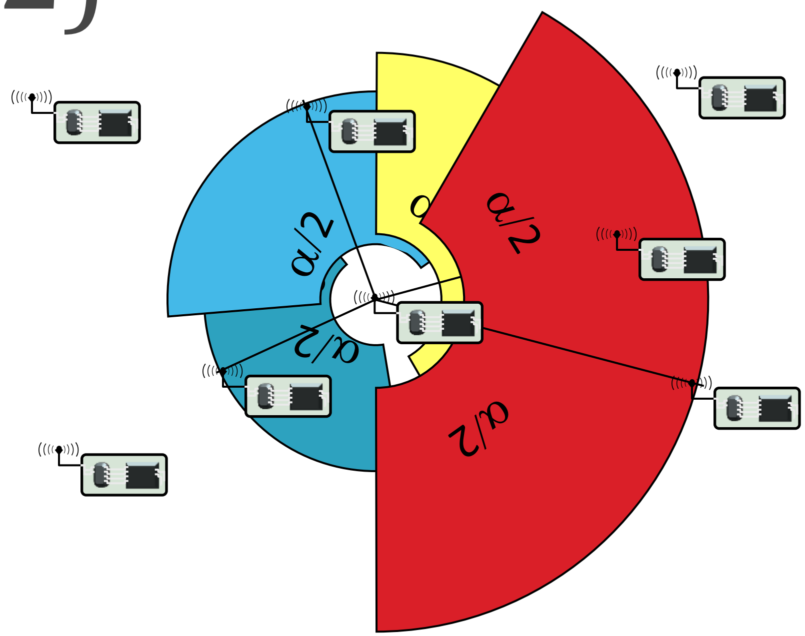
- Assign, to each node, all points in the plane for which it is the closest node
  - ! ***Voronoi diagram***
    - Constructed in O(|V| log |V|) time
- Connect any two nodes for which the Voronoi regions touch
  - ! ***Delaunay triangulation***
- Problem: Might produce very long links; not well suited for power control



Example: Cone-based topology control

- Assumption: Distance and angle information between nodes is available
- Two-phase algorithm
- Phase 1
  - Every node starts with a small transmission power
  - Increase it until a node has sufficiently many neighbors
  - What is “sufficient”? – When there is at least one neighbor in each ***cone*** of angle α
  - $\alpha = 5/6\pi$ is necessary and sufficient condition for connectivity!
- Phase 2
  - Remove redundant edges: Drop a neighbor w of u if there is a node v of w and u such that sending from u to w directly is less efficient than sending from u via v to w
  - Essentially, a local Gabriel graph construction

Example: Cone-based topology control (2)



- Properties: simple, local construction
- Extensions for k-connectivity (Yao graph)

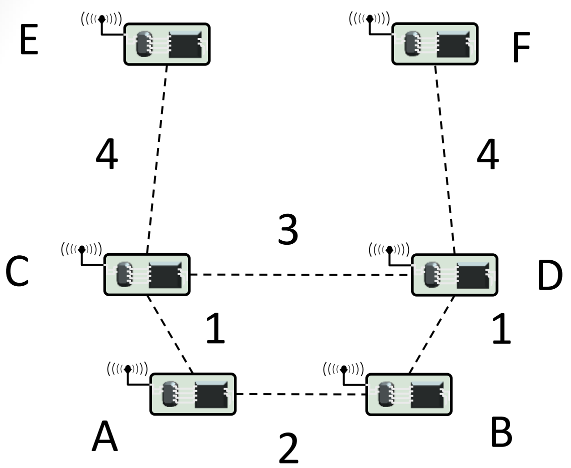
- Little exercise: What happens when α < or > 5/6 π?

Centralized power control algorithm

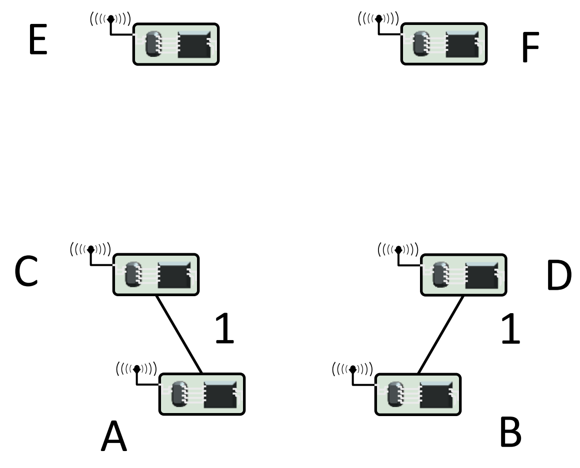
- Goal: Find topology control algorithm minimizing the ***maximum*** power used by any node
  - Ensuring simple or bi-connectivity
  - Assumptions: Locations of all nodes and path loss between all node pairs are known; each node uses an individually set power level to communicate with all its neighbors
- Idea: Use a centralized, greedy algorithm
  - Initially, all nodes have transmission power 0
  - Connect those two components with the shortest distance between them (raise transmission power accordingly)
- Second phase: Remove links (=reduce transmission power) not needed for connectivity
- Exercise: Relation to Kruskal's MST algorithm?

Centralized power control algorithm

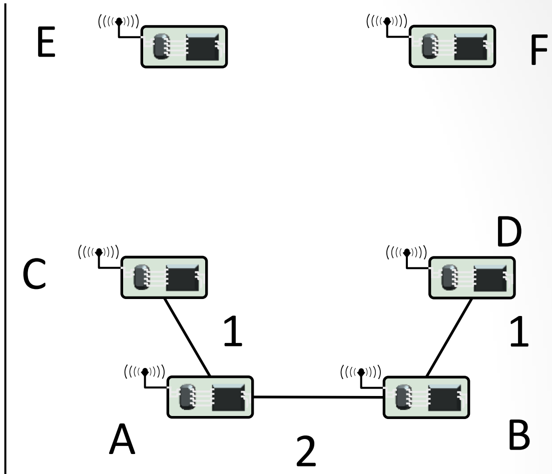
Topology



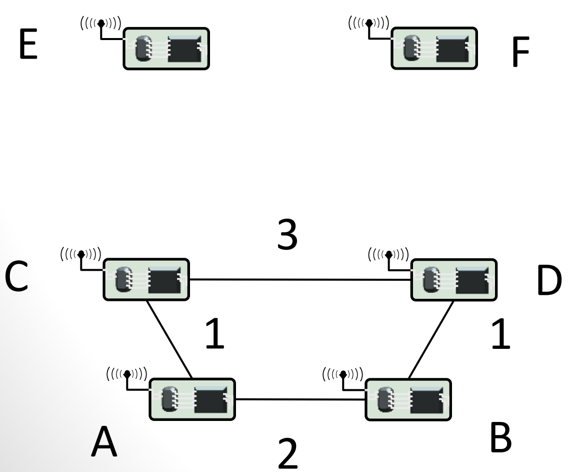
1) Connect A-C and B-D



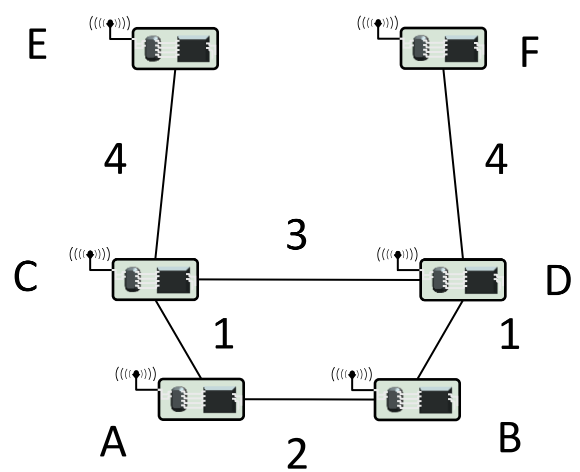
2) Connect A-B



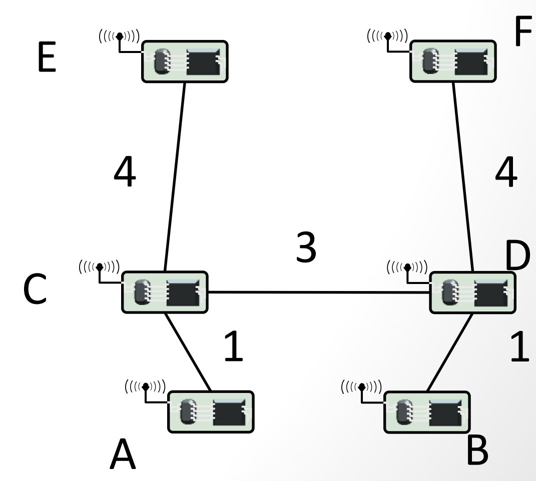
3) Connect C-D



4) Connect C-E and D-F



5) Remove edge A-B



BACKBONE CONSTRUCTION

Hierarchical networks – backbones

- Idea: Select some nodes from the network/graph to form a ***backbone***
  - A connected, minimal, dominating set (MDS or MCDS)
  - Dominating nodes control their neighbors
  - Protocols like routing are confronted with a simple topology – from a simple node, route to the backbone, routing in backbone is simple (few nodes)

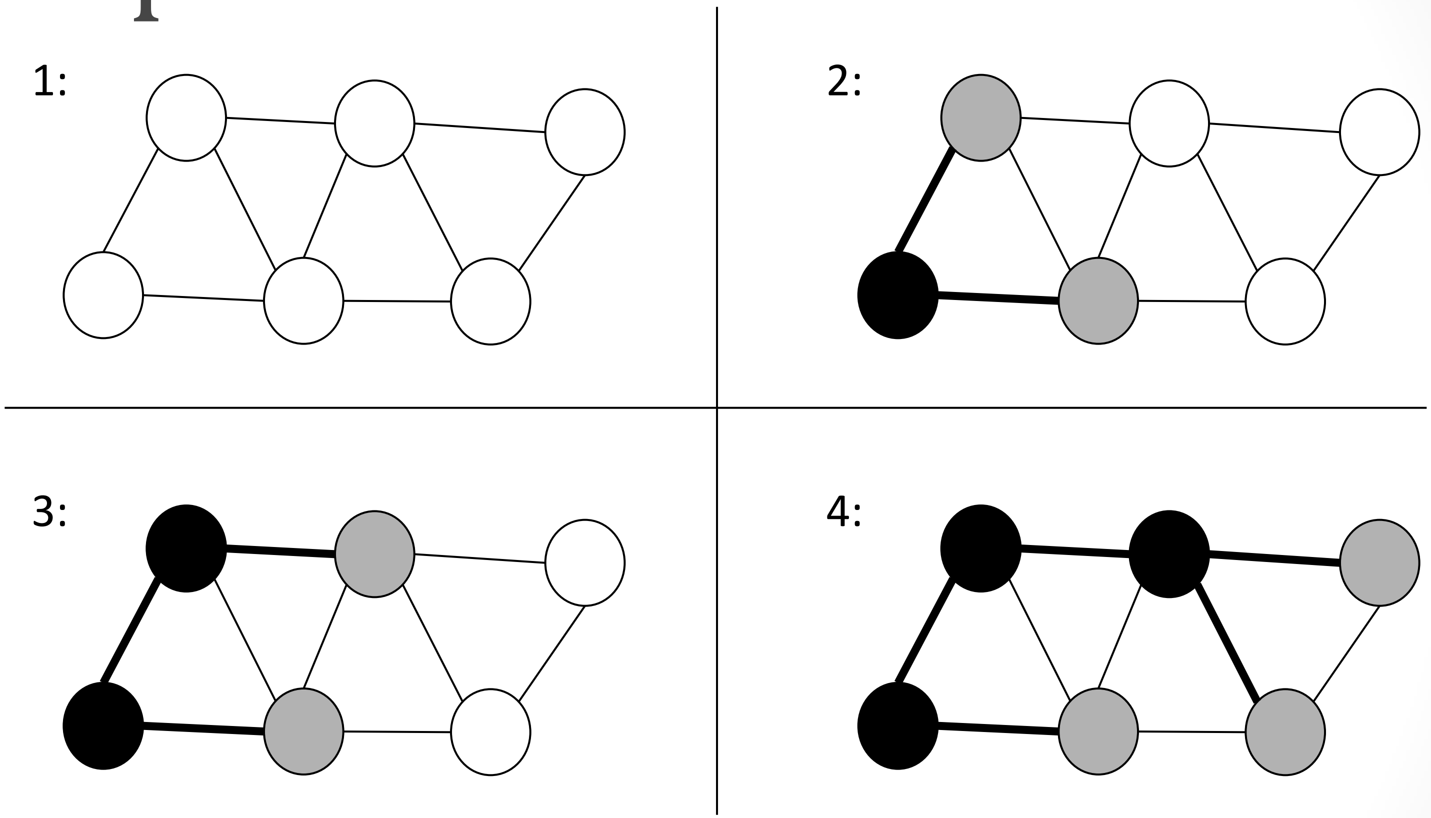
- Problem: MDS is an NP-hard problem
  - Hard to approximate, and even approximations need quite a few messages

Backbone by growing a tree

- Construct the backbone as a tree, grown iteratively

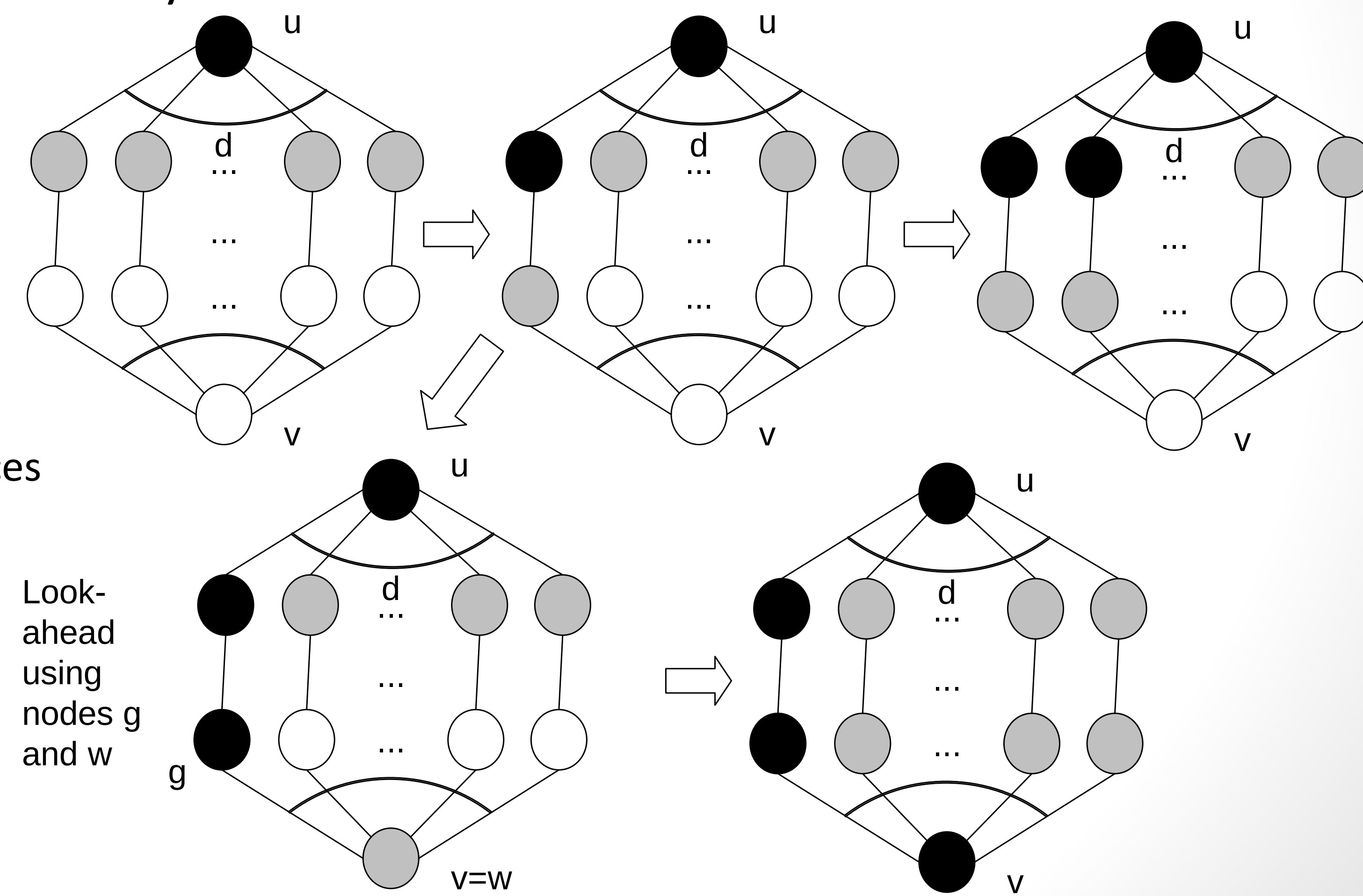
```
initialize all nodes' color to white
pick an arbitrary node and color it grey

while (there are white nodes) {
  pick a grey node v that has white neighbors
  color the grey node v black
  foreach white neighbor u of v {
    color u grey
    add (v,u) to tree T
  }
}
```

Backbone by growing a tree – Example



Problem: Which gray node to pick?

- When blindly picking any gray node to turn black, resulting tree can be very bad



Solution:
Look ahead!
One step suffices

Performance of tree growing with look ahead

- Dominating set obtained by growing a tree with the look ahead heuristic is at most a factor $2(1+ H(\Delta))$ larger than MDS
  - H(¢) harmonic function, $H(k) = \sum_{i=1}^{k} 1/i <= \ln k + 1$
  - Δ is maximum degree of the graph

- It is automatically connected

- Can be implemented in a distributed fashion as well

Start big, make lean

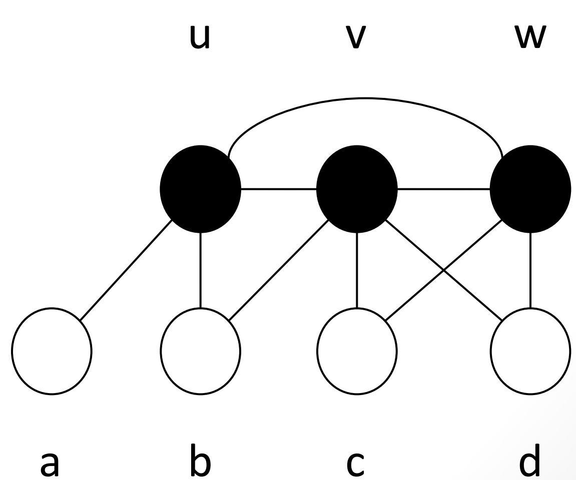
- Idea: start with some, possibly large, connected dominating set, reduce it by removing unnecessary nodes
- Initial construction for dominating set
  - All nodes are initially white
  - Mark any node black that has two neighbors that are not neighbors of each other (they might need to be dominated)

  ! Black nodes form a connected dominating set (proof by contradiction); shortest path between ANY two nodes only contains black nodes

- Needed: Pruning heuristics

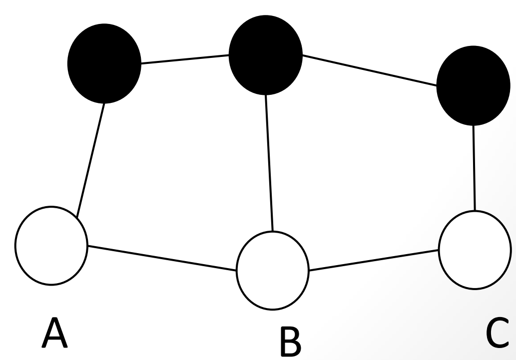
Pruning heuristics

- Heuristic 1: Unmark node v if
  - Node v and its neighborhood are included in the neighborhood of some node marked node u (then u will do the domination for v as well)
  - Node v has a smaller unique identifier than u (to break ties)
- Heuristic 2: Unmark node v if
  - Node v's neighborhood is included in the neighborhood of two marked neighbors u and w
  - Node v has the smallest identifier of the tree nodes
- Nice and easy, but only linear approximation factor



One more distributed backbone heuristic: Span

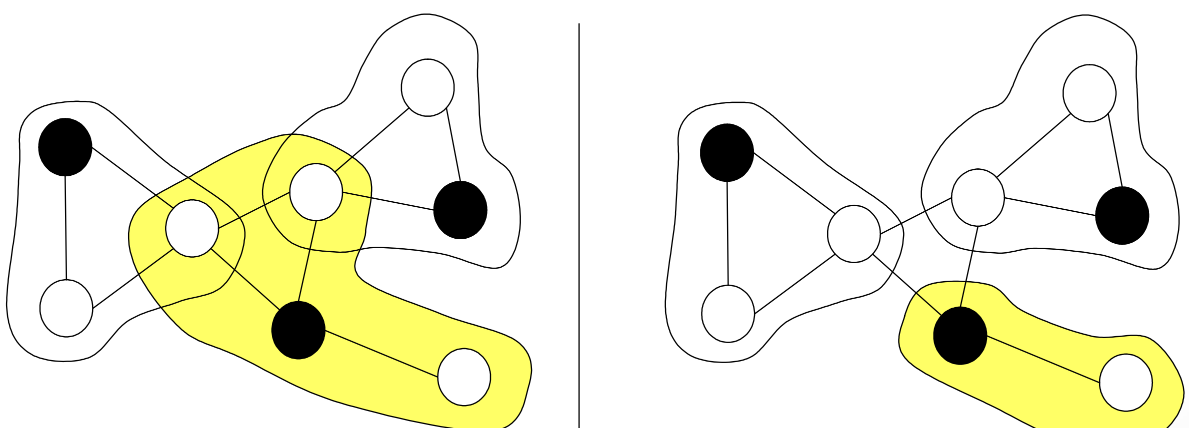
- Construct backbone, but take into account need to carry traffic – preserve capacity
  - Means: If two paths could operate without interference in the original graph, they should be present in the reduced graph as well
  - Idea: If the stretch factor (induced by the backbone) becomes too large, more nodes are needed in the backbone
- Rule: Each node observes traffic around itself
  - If node detects two neighbors that need three hops to communicate with each other, node joins the backbone, shortening the path
  - Contention among potential new backbone nodes handled using random backoff



CLUSTERING

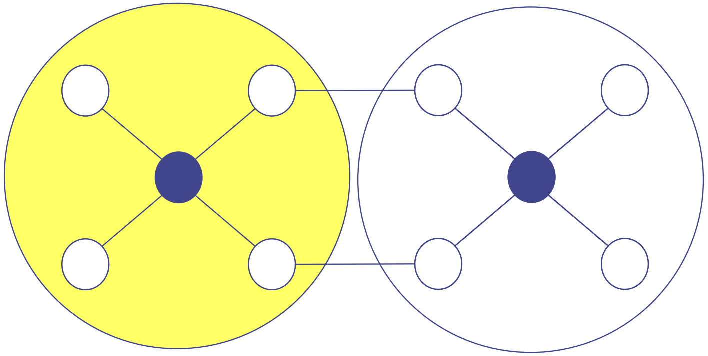
Clustering

- Partition nodes into groups of nodes – ***clusters***
- Many options for details
  - Are there ***clusterheads***? – One controller/representative node per cluster
  - May clusterheads be neighbors? If no: clusterheads form an ***independent set C:*** $\forall c_1, c_2 \in C : (c_1, c_2) \notin E$
    Typically: clusterheads form a ***maximum independent set***
  - May clusters overlap? Do they have nodes in common?



Clustering

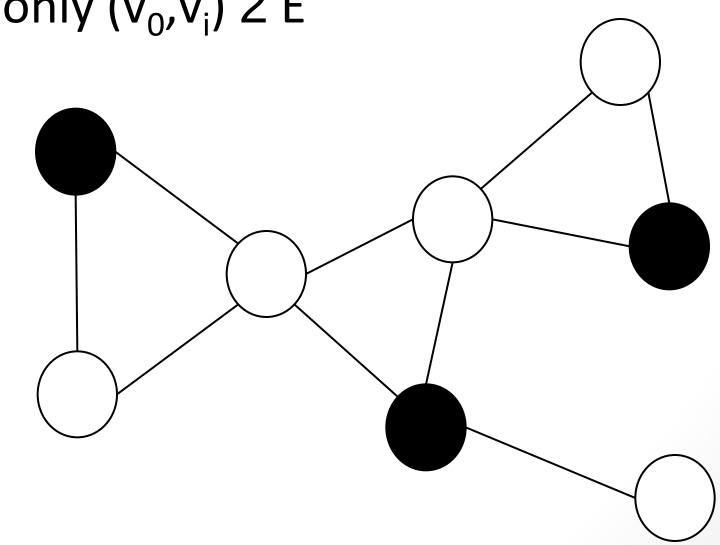
- Further options
  - How do clusters communicate? Some nodes need to act as ***gateways*** between clusters
    If clusters may not overlap, two nodes need to jointly act as a ***distributed gateway***



  - How many gateways exist between clusters? Are all active, or some standby?
  - What is the maximal diameter of a cluster? If more than 2, then clusterheads are not necessarily a maximum independent set
  - Is there a hierarchy of clusters?

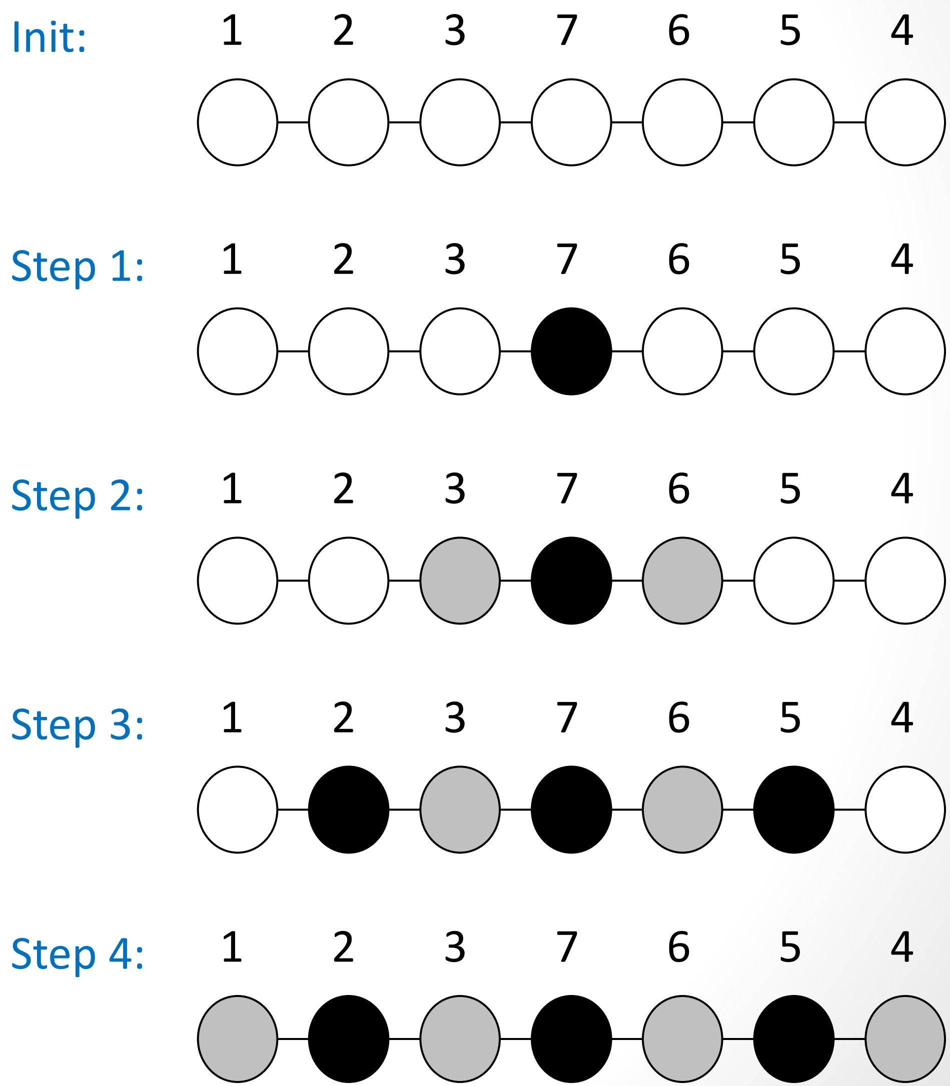
Maximum independent set

- Computing a maximum independent set is NP-complete
- Can be approximate within $(\Delta +3)/5$ for small Δ, within $O(\Delta \log \log \Delta / \log \Delta)$ else; Δ bounded degree
- Show: A maximum independent set is also a dominating set
- Maximum independent set not necessarily intuitively desired solution
  - Example: Radial graph, with only (v_0,v_i) 2 E



A basic construction idea for independent sets

- Use some attribute of nodes to break local symmetries
  - Node identifiers, energy reserve, mobility, weighted combinations… - matters not for the idea as such (all types of variations have been looked at)
- Make each node a clusterhead that locally has the largest attribute value
- Once a node is dominated by a clusterhead, it abstains from local competition, giving other nodes a chance



Determining gateways to connect clusters

- Suppose: Clusterheads have been found
- How to connect the clusters, how to select gateways?

- It suffices for each clusterhead to connect to all other clusterheads that are at most three hops
  - Resulting backbone (!) is connected

- Formally: Steiner tree problem
  - Given: Graph G=(V,E), a subset C ½ V
  - Required: Find another subset T ½ V such that S [T is connected and S [T is a cheapest such set
  - Cost metric: number of nodes in T, link cost
  - Here: special case since C are an independent set

Rotating clusterheads

- Serving as a clusterhead can put additional burdens on a node
  - For MAC coordination, routing, …

- Let this duty rotate among various members
  - Periodically reelect – useful when energy reserves are used as discriminating attribute
  - LEACH – determine an optimal percentage P of nodes to become clusterheads in a network
    - Use 1/P rounds to form a period
    - In each round, nP nodes are elected as clusterheads
    - At beginning of round r, node that has not served as clusterhead in this period becomes clusterhead with probability P/(1-p(r mod 1/P))

Multi-hop clusters

- Clusters with diameters larger than 2 can be useful, e.g., when used for routing protocol support
- Formally: Extend “domination” definition to also dominate nodes that are at most d hops away
- Goal: Find a smallest set D of dominating nodes with this extended definition of dominance
- Only somewhat complicated heuristics exist

- Different tilt: Fix the ***size*** (not the diameter) of clusters
  - Idea: Use ***growth budgets*** – amount of nodes that can still be adopted into a cluster, pass this number along with broadcast adoption messages, reduce budget as new nodes are found

Passive clustering

- Constructing a clustering structure brings overheads
  - Not clear whether they can be amortized via improved efficiency
- Question: Eat cake and have it?
  - Have a clustering structure without any overhead?
  - Maybe not the best structure, and maybe not immediately, but benefits at zero cost are no bad deal…

! Passive clustering

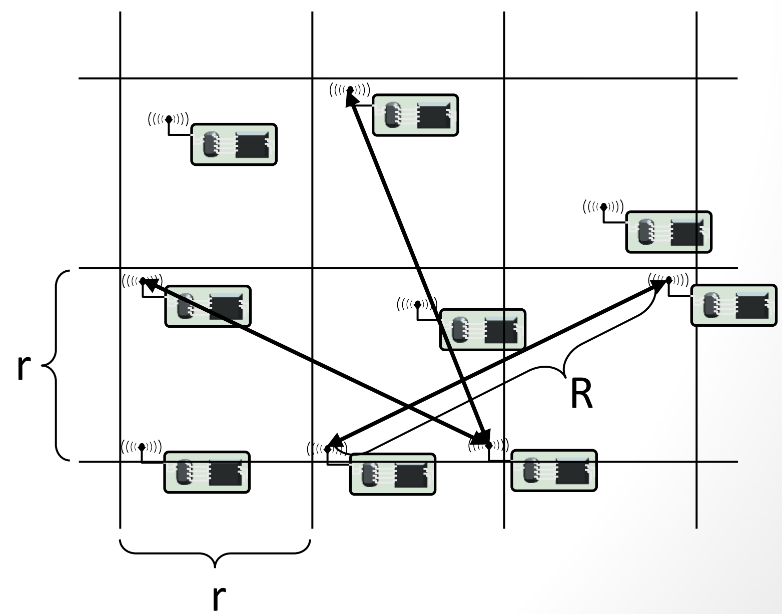
  - Whenever a broadcast message travels the network, use it to construct clusters on the fly
  - Node to start a broadcast: Initial node
  - Nodes to forward this first packet: Clusterhead
  - Nodes forwarding packets from clusterheads: ordinary/gateway nodes
  - And so on… ! Clusters will emerge at low overhead

ADAPTIVE NODE ACTIVITY

Adaptive node activity

- Remaining option: Turn some nodes off deliberately
- Only possible if other nodes remain on that can take over their duties
- Example duty: Packet forwarding
  - Approach: Geographic Adaptive Fidelity (GAF)

- Observation: Any two nodes within a square of length $r < R/5^{1/2}$ can replace each other with respect to forwarding
  - R radio range
- Keep only one such node active, let the other sleep



Conclusion

- Various approaches exist to trim the topology of a network to a desired shape
- Most of them bear some non-negligible overhead
  - At least: Some distributed coordination among neighbors, or they require additional information
  - Constructed structures can turn out to be somewhat brittle – overhead might be wasted or even counter-productive
- Benefits have to be carefully weighted against risks for the particular scenario at hand

- For more details, refer to:
  - Chapter 10, H. Karl and A. Willig, **Protocols and Architectures for Wireless Sensor Networks**, Wiley 2005.
- The lecture is available online at:
  - http://bu.edu.eg/staff/ahmad.elbanna-courses/12189
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